RSP Measurement Model

In the analysis of RSP data both absolute measurements, such as the radiance *I* and polarized radiance *Q*, and relative quantities such as *q=Q/I* are used. A model for how these measurements are constructed from the signals in each channel and the various calibration coefficients is given by

$$
\begin{aligned}
I &= \alpha_c \left(K1^{-0.5} S1L + K1^{0.5} S1R \right) \\
Q &= \alpha_c \alpha_1 \left(K1^{-0.5} S1L - K1^{0.5} S1R \right) \\
q &= \alpha_1 \left(\frac{K1^{-0.5} S1L - K1^{0.5} S1R}{K1^{-0.5} S1L + K1^{0.5} S1R} \right)
\end{aligned}
\qquad (1)
$$

in which the application of the K1 coefficients has been symmetrized and it is assumed that S1L and S1R are calibrated such that the mean values of these calibration coefficients are unity. This allows us to focus on what the effects of errors in the calibration coefficients are. α_c and $\alpha 1$ are the absolute radiometric calibration coefficient and the polarimetric calibration coefficient and since they, and the relative gain coefficients K1, are scale variables we assume they are log normally distributed. In order to construct a noise model we note that for any function $f(\mathbf{x})$ where $\mathbf{x}=(x_1, x_2, \ldots\ldots x_n)$ to first order the uncertainty in that function is given by

$$
\sigma_f^2 = \sum_{i=1}^{n} \left[\frac{\partial f(\mathbf{x})}{\partial x_i} \right]^2 \sigma_i^2 \qquad (2)
$$

The partial derivatives required in the calculation of the uncertainty in *I* and *Q* are given by the following formulae

$$
\begin{aligned}
\frac{\partial I}{\partial S1L} &= \alpha_c K1^{-0.5} = 1 \\
\frac{\partial I}{\partial S1R} &= \alpha_c K1^{0.5} = 1 \\
\frac{\partial I}{\partial \ln K1} &= \frac{-\alpha_c}{2} \left(K1^{-0.5} S1L - K1^{0.5} S1R \right) = \frac{-Q}{2} \\
\frac{\partial I}{\partial \ln \alpha_c} &= \alpha_c \left(K1^{-0.5} S1L + K1^{0.5} S1R \right) = I
\end{aligned}
\qquad (3)
$$

and

$$
\begin{aligned}
\frac{\partial Q}{\partial S1L} &= \alpha_c \alpha 1 K1^{-0.5} = 1 \\
\frac{\partial Q}{\partial S1R} &= \alpha_c \alpha 1 K1^{+0.5} = -1 \\
\frac{\partial Q}{\partial \ln K1} &= \frac{-\alpha_c \alpha_1}{2}\left(K1^{-0.5} S1L + K1^{0.5} S1R\right) = -\frac{I}{2} \\
\frac{\partial Q}{\partial \ln \alpha 1} &= \alpha_c \alpha_1 \left(K1^{-0.5} S1L - K1^{0.5} S1R\right) = Q \\
\frac{\partial Q}{\partial \ln \alpha_c} &= \alpha_c \alpha 1 \left(K1^{-0.5} S1L - K1^{0.5} S1R\right) = Q
\end{aligned}
\tag{4}
$$

If we note that

$$
\frac{\partial q}{\partial x_i} = \frac{1}{I}\left(\frac{\partial Q}{\partial x_i} - q\frac{\partial I}{\partial x_i}\right) \tag{5}
$$

then the partial derivatives required in the evaluation of *q* can be derived from eqs. (3) and (4) and are given by

$$
\begin{aligned}
\frac{\partial q}{\partial S1L} &= \frac{1-q}{I} \\
\frac{\partial q}{\partial S1R} &= -\frac{1+q}{I} \\
\frac{\partial q}{\partial \ln K1} &= -\frac{\left(1-q^2\right)}{2} \\
\frac{\partial q}{\partial \ln \alpha 1} &= q
\end{aligned}
\tag{6}
$$

A model for detector noise that is generally valid for any system is

$$
\sigma_{noise}^2 = \sigma_{floor}^2 + aI_{channel} \tag{7}
$$

where $I_{channel}$ is the intensity in the given channel, $aI_{channel}$ is the shot noise contribution and the subscript "floor" contains the contributions from dark noise, readout noise etc. Using this model for noise together with the recognition that $S1L=(I+Q)/2$ and $S1R=(I-Q)/2$ we obtain the following expression for uncertainties in the intensity caused by noise and calibration uncertainties viz.,

$$
\begin{aligned}
\sigma_I^2(noise) &= 2\sigma_{floor}^2 + aI \\
\sigma_I^2(calibration) &= \sigma_{\ln K1}^2 \frac{Q^2}{4} + \sigma_{\ln \alpha_c}^2 I^2
\end{aligned}
\tag{8}
$$

where this is the expression for the intensity in the telescopes that measure *Q* and a similar expression applies to the telescopes measuring *U*. Given that the uncertainty in K1 is less than 0.1% and is calibrated continuously whereas the absolute radiometric calibration uncertainty is between 2-5% the contribution of uncertainties in K1 to the radiometric uncertainty is negligibly small even if the DoLP is 100%. The uncertainties in the Stokes parameter *Q* are given by the expression

$$\begin{aligned} \sigma_Q^2(noise) &= 2\sigma_{floor}^2 + aI \\ \sigma_Q^2(calibration) &= \sigma_{\ln K1}^2 \frac{I^2}{4} + \left(\sigma_{\ln \alpha_c}^2 + \sigma_{\ln \alpha 1}^2\right) Q^2 \end{aligned} \tag{9}$$

which shows the expected behavior that uncertainties in Q caused by calibration are generally dominated by the absolute calibration except when $q<2\%$ in which case uncertainties in the K1 coefficient start to become significant. The uncertainties in the normalized Stokes parameter *q* are then

$$\begin{aligned} \sigma_q^2(noise) &= \frac{2(1+q^2)}{I^2}\sigma_{floor}^2 + \frac{a(1-q^2)}{I} \\ \sigma_q^2(calibration) &= \sigma_{\ln K1}^2 \frac{\left(1-q^2\right)^2}{4} + \sigma_{\ln \alpha 1}^2 q^2 \end{aligned} \tag{10}$$

The somewhat surprising consequence of the first of these equations is that shot noise has no effect when $q=\pm1$. This is correct and is the result of the fact that in this case one channel is dark and the other channel has all the signal so any shot noise contributions are cancelled out by the division of signal values that generates *q*. The variance of the DoLP is simply the sum of the variances in the normalized Stokes parameters *q* and *u* viz.,

$$\sigma_{DoLP}^2 = \sigma_q^2 + \sigma_u^2 \tag{11}$$

The uncertainty due to noise is then

$$\sigma_{DoLP}^2(noise) = \frac{2\sigma_{floor}^2}{I^2}(2+q^2+u^2) + \frac{a}{I}(2-q^2-u^2) \tag{12}$$

which can be simplified to

$$\sigma_{DoLP}^2(noise) = \frac{4\sigma_{floor}^2}{I^2}\left(1+\frac{DoLP^2}{2}\right) + \frac{2a}{I}\left(1-\frac{DoLP^2}{2}\right). \tag{12}$$

The uncertainty due to calibration is

$$\sigma^2_{DoLP}(calibration) = \frac{\sigma^2_{\ln K1}}{4}\left(1-2q^2+q^4\right)+\sigma^2_{\ln\alpha1}q^2+\frac{\sigma^2_{\ln K2}}{4}\left(1-2u^2+u^4\right)+\sigma^2_{\ln\alpha2}u^2$$
(13)

which, if the uncertainties in *K1* and *K2* are similar and the uncertainties in *α1* and *α2* are similar (i.e. $\sigma_{\ln_{\alpha}1}= \sigma_{\ln_{\alpha}1}= \sigma_{\ln_{\alpha}}$ and $\sigma_{\ln K1}= \sigma_{\ln K2}= \sigma_{\ln K}$), as is the case for all RSP and APS measurement channels) can be simplified to

$$\sigma^2_{DoLP}(calibration) = \frac{\sigma^2_{\ln K}}{2}\left(1-DoLP^2\right)+\sigma^2_{\ln\alpha}DoLP^2+\frac{\sigma^2_{\ln K}}{4}(q^4+u^4) \quad (14)$$

The last term can be simplified somewhat if we recognize that *q=DoLP.cos(2χ)* and *u=DoLP.sin(2χ)* where χ is the polarization azimuth, in which case

$$\begin{aligned}
(q^4+u^4) &= DoLP^4\left(\cos^4 2\chi+\sin^4 2\chi\right)\\
(q^4+u^4) &= DoLP^4\left(\cos^2 2\chi(1-\sin^2 2\chi)+\sin^2 2\chi(1-\cos^2 2\chi)\right)\\
(q^4+u^4) &= DoLP^4\left(1-2\sin^2 2\chi\cos^2 2\chi\right)\\
(q^4+u^4) &= DoLP^4\left(1-\frac{1}{2}\left(2\sin 2\chi\cos 2\chi\right)^2\right)\\
(q^4+u^4) &= DoLP^4\left(1-\frac{1}{2}\sin^2 4\chi\right)
\end{aligned} \quad (15)$$

and

$$\sigma^2_{DoLP}(calibration) = \frac{\sigma^2_{\ln K}}{2}\left[1-DoLP^2+\frac{DoLP^4}{2}\left(1-\frac{1}{2}\sin^2 4\chi\right)\right]+\sigma^2_{\ln\alpha}DoLP^2 \quad (16)$$

We now recall that

$$I = \frac{\mu_s R F_0}{\pi r} \quad (17)$$

where F_0 is the solar constant, *r* is the solar distance in astronomical units and μ_s is the cosine of the solar zenith angle and introduce the parameters $a'=a\pi/F_0$ and $\sigma'_{floor}=\sigma_{floor}\pi/F_0$, which scale the noise model parameters from the dimension of radiance to normalized radiance. The uncertainties due to noise can then be written in terms of reflectances and the final result for the variance caused by noise in terms of the DoLP is

$$\sigma^2_{DoLP}(noise) = 4\left(1+\frac{DoLP^2}{2}\right)\left(\frac{r^2\sigma'_{floor}}{\mu_s R_I}\right)^2+2\left(1-\frac{DoLP^2}{2}\right)\frac{a'r^2}{\mu_s R_I} \quad (17)$$

.

For instruments like APS and RSP the intensities in the two telescopes are observations of the same scene and a small increase in SNR and a symmetrical polarization impact on intensity is obtained by averaging these two intensities together. This obviously does not reduce the radiometric uncertainty because the absolute gain for each telescope has the same calibration path. The uncertainty in this average total intensity caused by noise and calibration are then given by

$$\begin{aligned} \sigma_{\bar{I}}^2(noise) &= \sigma_{floor}^2 + \frac{a}{2}\bar{I} \\ \sigma_{\bar{I}}^2(calibration) &= \frac{\sigma_{\ln K}^2}{16}\left(Q^2 + U^2\right) + \sigma_{\alpha_c}^2 \bar{I}^2 \end{aligned} \quad (18)$$

Similarly we are often interested in the polarized intensity, I_p, which is given by the expression

$$I_p = \sqrt{Q^2 + U^2} \quad (19)$$

and has the noise and calibration related uncertainties

$$\begin{aligned} \sigma_{I_p}^2(noise) &= 2\left(2\sigma_{floor}^2 + \bar{I}\right) \\ \sigma_{I_p}^2(calibration) &= \frac{\sigma_{\ln K}^2}{2}\bar{I}^2 + \left(\sigma_{\alpha_c}^2 + \sigma_{\alpha}^2\right) I_p \end{aligned} \quad (20)$$

The uncertainty in the total reflectance is then given by

$$\begin{aligned} \sigma_{R_I}^2(noise) &= \left(\frac{r^2 \sigma'_{floor}}{\mu_s}\right)^2 + \frac{a'}{2}\frac{R_I r^2}{\mu_s} \\ \sigma_{R_I}^2(calibration) &= \frac{\sigma_{\ln K}^2}{16} R_P^2 + \sigma_{\alpha_c}^2 R_I^2 \end{aligned} \quad (21)$$

and the uncertainty in the polarized reflectance is

$$\begin{aligned} \sigma_{R_P}^2(noise) &= 4\left(\frac{r_\circ^2 \sigma'_{floor}}{\mu_s}\right)^2 + 2a'\frac{R_I r_\circ^2}{\mu_s} \\ \sigma_{R_P}^2(calibration) &= \frac{\sigma_{\ln K}^2}{2} R_I^2 + \left(\sigma_{\alpha_c}^2 + \sigma_{\alpha}^2\right) R_P^2 \end{aligned} \quad (21)$$

The values for the noise floor and shot noise parameters for APS are given in the following table in units of steradian^{-1}. This is the unit of a normalized radiance and is used so that the significant variations in radiance across the 412-2260 nm range are reduced and allow the use of a single pair of values for simple analysis. In practice the noise floor is measure on every scan for APS and RSP while the shot noise contribution is modeled and the model verified against radiometric calibration measurements. Deviations of a complete noise model (that includes Johnson noise, boost noise, ADC noise, shot noise etc.) from the simple floor and shot noise model given here are of order 0.1% and are therefore negligible.

Band	410	443	555	670	865	910	1378	1610	2250
a'	6.9E-8	5.6E-8	3.7E-8	3.7E-8	2.3E-8	4.4E-8	1.6E-8	1.2E-8	2.3E-8
σ'_{floor}	6.9E-5	5.7E-5	4.0E-5	4.1E-5	3.0E-5	4.6E-5	2.0E-5	1.9E-5	2.8E-5

For RSP similar values are found viz.,

Band	410	470	555	670	865	960	1590	1880	2260
σ'_{floor}	3.2E-05	2.5E-05	2.4E-05	2.2E-05	2.0E-05	2.1E-05	1.8E-05	1.8E-05	1.9E-05
a'	2.3E-08	1.2E-08	4.5E-09	3.7E-09	3.7E-09	6.8E-09	1.8E-08	6.6E-09	8.2E-09

The effects of the noise floor can therefore be captured conservatively using σ'_{floor}=1E-4 and similarly the effects of shot noise can be captured using a'=1E-7 at least for RSP and APS type instruments.

K1 and K2 values can vary by ±0.5% over environments and between field experiments but are determined using the inflight calibrators for APS (and RSP) to within ±0.05% for each file. APS had an inflight calibrator for tracking the α1 and α2 values, which would have allowed them to be determined to within ±0.05%. For RSP a more appropriate value for the uncertainty between calibrations is ±0.1%, since calibrations are usually performed pre- and post- deployment. For example a worst case is given by the differences in the table below, which is between calibrations performed in 05/2005 and 12/2010.

Band	1	2	3	4	5	6	7	8	9
$\sigma_{\alpha q}$	-0.06%	0.01%	0.18%	0.13%	0.11%	0.12%	-0.03%	-0.14%	-0.21%
$\sigma_{\alpha u}$	-0.13%	-0.01%	0.00%	0.00%	0.03%	0.01%	-0.11%	-0.10%	-0.09%

Uncertainty in radiometric calibration for reflectance would be ~2-3% for an instrument like APS where a well characterized reflector is used as an initial injection reflectance standard that is then transferred to the moon. For RSP, which uses integrating spheres and which are a secondary (or even tertiary) standard after transfer from a standard lamp the radiometric uncertainty is ~3-5% because of lamp uncertainties and uncertainties in the solar spectral irradiance.

The variation in response versus scan for an instrument such as APS or RSP is generally quite small because the mirrors in the scanner are a matched pair that is fabricated in the same coating run. The measured variations in response versus scan (standard deviations over the angular range from +50.5/-62°) are given in the following table. Angle to angle uncertainties are therefore far smaller than for instruments such as POLDER or MISR because the same detectors observe all view angles and the variation in throughput of the mirrors as a function of angle is very small because they are operated with constant angle of incidence, unlike the MODIS scan mechanism.

Band	1	2	3	4	5	6	7	8	9
RVS_i1	0.16%	0.12%	0.11%	0.15%	0.21%	0.21%	0.19%	0.14%	0.25%
RVS_i2	0.17%	0.28%	0.09%	0.18%	0.22%	0.21%	0.24%	0.21%	0.26%
RVS_p	0.09%	0.08%	0.04%	0.06%	0.07%	0.08%	0.09%	0.12%	0.09%